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Remarks on abelian dominance ¹

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Abstract

We used a renormalisation group based smoothing to address two questions related to abelian dominance. Smoothing drastically reduces short distance fluctuations but it preserves the long distance physical properties of the $SU(2)$ configurations. This enabled us to extract the abelian heavy-quark potential from time-like Wilson loops on Polyakov gauge projected configurations. We obtained a very small string tension which is inconsistent with the string tension extracted from Polyakov loop correlators. This shows that the Polyakov gauge projected abelian configurations do not have a consistent physical meaning. We also applied the smoothing on $SU(2)$ configurations to test how sensitive abelian dominance in the maximal abelian gauge is to the short distance fluctuations. We found that on smoothed $SU(2)$ configurations the abelian string tension was about 30% smaller than the $SU(2)$ string tension which was unaffected by smoothing. This suggests that the approximate abelian dominance found with the Wilson action is probably an accident and it has no fundamental physical relevance.

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1 Introduction

It is an old idea to try to understand non-abelian gauge theories in terms of an effective abelian model with a smaller symmetry group. One possible way of doing this on the lattice is to isolate $U(1)^{N-1}$ link variables belonging to a maximal torus of $SU(N)$. This is called abelian projection. The hope is that non-abelian confinement might be explained as a condensation of monopoles in the resulting abelian projected model (see e.g. [1] for a recent review). If one wants to explain the non-abelian physics in the abelian projected system, a necessary condition is that the abelian model has to reproduce the physical features of the non-abelian system. This property is referred to as abelian dominance.

Since the abelian symmetry group is smaller than the non-abelian one, this procedure necessarily involves some gauge fixing before the projection is done. In principle the physical properties of the projected system can depend on the gauge choice. This was originally perceived as a nuisance and there are still attempts to prove that the physical properties of the projected system are independent of the gauge choice (see e.g. [2]). Up to now the only gauge in which abelian projected system seems to capture the physics of the non-abelian model is the maximal abelian gauge [3]. Here in the $SU(2)$ case the abelian and non-abelian string tensions at Wilson $\beta = 2.51$ agree to within 8% [4]. In other gauges, most notably in the Polyakov gauge (where Polyakov loops are diagonalised) the situation is more controversial. Since all the Polyakov loops can be exactly diagonalised at the same time, in this case “abelian dominance” exactly and trivially holds if the string tension is measured with Polyakov loop correlators. On the other hand due to the high level of noise on the projected configurations, it is impossible to extract the string tension from Wilson loops [5, 6].

In the present paper we study some related issues. The first question we address is that of the gauge choice. We use a recently proposed smoothing technique based on renormalisation group ideas [7]. We can drastically reduce the short-distance fluctuations while preserving the long-distance physical properties of our configurations, most importantly the $SU(2)$ string tension. This allows us to extract the heavy quark potential from Wilson loops on Polyakov gauge projected configurations. Doing the gauge fixing and the projection on the smoothed configurations, the resulting abelian string tension turns out to be practically zero. This result is inconsistent with the string tension measured from Polyakov loop correlators. It shows that the physical meaning of Polyakov gauge projected configurations is questionable. We argue on general grounds that probably some other gauge choices also suffer from the same problem and we also present a set of minimal requirements that a gauge choice has to satisfy in order to avoid this problem. We conclude that it is natural to expect

that the gauge choice has a strong influence on the physical properties of the abelian configurations. We suggest that rather than trying to prove the gauge independence of the projection, one has to concentrate on finding one particular gauge in which abelian dominance holds. The only gauge known to us in which approximate abelian dominance has been found (with the Wilson action) is the maximal abelian one. Therefore in the second part of the present paper we shall concentrate only on this gauge.

The approximate nature of the agreement between the abelian and non-abelian string tension raises some doubts as to whether it is a real physical effect or an accident. In particular, if abelian dominance is a genuine physical effect, it should hold in the continuum limit and also it should be universal. We can look at the abelian and non-abelian string tensions as two different physical quantities with the same mass dimension. Their ratio in the continuum limit has to be unique and if abelian dominance holds, close to unity. Unfortunately a detailed scaling test on any abelian quantity would be extremely expensive in an iteratively fixed gauge like the maximal abelian one. In the present work we have a much more modest aim. We study a related question, how abelian dominance depends on the details of the short-distance fluctuations. Using the above mentioned smoothing on Monte Carlo generated $SU(2)$ gauge configurations we can produce smoothed configurations with the same long-distance properties but reduced short-distance fluctuations. Comparing the abelian string tension on the original and the smoothed configurations we can gain insight into its dependence on the short-distance details. We find that the abelian string tension is very sensitive to the short distance structure dropping by about 30% after one smoothing step. This raises the question of how much of the abelian string tension comes from genuine $SU(2)$ long distance physics and how much of it is a reflection of physically irrelevant short range fluctuations.

The plan of the paper is as follows. In Section 2 we briefly describe the smoothing procedure that will be used. In Section 3 we address the question of the gauge choice, present our results about the string tension on Polyakov gauge projected configurations and make some general remarks. In Section 4 we study the question of universality of abelian dominance in the maximally abelian gauge. Finally in Section 5 we present our conclusions.

2 Smoothing

In this section we describe the main idea of the smoothing procedure that we use. A more detailed account can be found in Ref. [7]. Let us consider a scale two real space renormalisation group transformation (blocking) that maps the original

(fine) lattice on a coarser lattice with twice the lattice spacing and 2^4 (in $d = 4$) times less degrees of freedom. By construction blocking preserves all physical features (correlations) of the fine lattice on distance scales larger than the coarse lattice spacing. Since blocking is a coarse graining procedure and there are many fine configurations which are mapped on the same coarse one, it has no inverse. Nevertheless one can define an opposite operation, we call it inverse blocking, which assigns to any coarse configuration the smoothest (smallest action) of all those fine configurations that block into it. Inverse blocking can be thought of as interpolating to a finer lattice in the smoothest possible way while preserving all the physical features of the coarse configuration. Now we can describe the smoothing procedure.

One step of smoothing consists of an inverse blocking on a finer lattice followed by a blocking but on a different coarse sublattice of the fine lattice. Using a different sublattice is essential because otherwise the fine configuration would just block back into the same coarse configuration that we started with. The crucial point is that if the original physical lattice spacing was a and the bare coupling $g(a)$ then the inverse blocked configurations will be locally a lot smoother than typical configurations at a coupling $g(a/2)$. This is because inverse blocking selects the lowest action configuration from among all the ones that would block into the given coarse configuration. Now a blocking step on a different coarse sublattice will treat the fine configuration as if it corresponded to a coupling larger than $g(a/2)$ and block it into a configuration with effective bare coupling larger than $g(a)$. This is however not true if the blocking is performed on the original coarse sublattice since in this case by construction we would get back the original configuration. This happens because the inverse blocked configuration has a certain staggered structure that “remembers” the fact that it came from a coarse lattice by inverse blocking, this is why a certain sublattice is distinguished.

The net result of one smoothing step is that the lattice returns to its original size, it has essentially unchanged long-distance physical content (since both blocking and inverse blocking preserve this) but the shortest distance fluctuations and consequently the action are considerably reduced. In Ref. [7] after a few such smoothing steps the action was seen to drop by almost two orders of magnitude while the string tension and the instanton content remained the same. On the other hand, the additive constant term of the heavy quark potential extracted from Wilson loops decreased considerably, reflecting the fact that much of the short distance fluctuations have been removed.

Performing several successive smoothing steps will gradually reduce fluctuations at larger and larger distance scales but it does not affect the genuine asymptotic long-distance observables. In Table 1 we illustrate how effective this smoothing procedure is in removing short-distance fluctuations. In this Table

we present how the average plaquette, the action and the string tension changes with the smoothing. The measurements were done on the same ensemble of $100 \times 8^3 \times 12$ fixed point action $\beta = 1.5$ configurations that we also use in Section 4.

Table 1: The average plaquette, the action and the string tension after smoothing. Step 0 refers to the original $8^3 \times 12$ configurations generated with a fixed point action at $\beta = 1.5$.

smoothing step	0	1	2	3
plaquette	1.030	1.908	1.960	1.972
action	35000	3100	1400	960
σa^2	0.123(7)	0.115(9)	0.112(7)	0.118(6)

The blocking kernel that we used is completely identical to the one of Ref. [7]. All of our configurations were generated with the fixed point action of Ref. [7] because this is the fixed point action corresponding to our RG transformation and therefore it is consistent with the blocking and inverse blocking.

3 The gauge choice

The very idea of abelian dominance is that the diagonal abelian degrees of freedom can account for the physical properties of the full non-abelian configurations. The issue of gauge fixing is definitely important here since the part of the system that we retain/discard with the abelian projection very strongly depends on it. There are several gauge fixing proposals in the literature in this context. The idea behind some of them is that a certain class of operators can be made diagonal by a suitable gauge fixing and for these operators the results obtained after abelian projection are exactly equal to the full non-abelian measurements. This is taken as evidence for abelian dominance (see e.g. [2] for the case of Polyakov loops in the Polyakov gauge).

We want to emphasise that this is only a trivial consequence of the non-abelian gauge freedom and it does not mean that the abelian part reproduces the relevant physical properties of the full system. On any given $SU(2)$ configuration all the links belonging to the Polyakov loops can be diagonalised simultaneously by a suitable gauge transformation. Therefore any physical quantity derived from Polyakov loops will be trivially and exactly reproduced after abelian projection in this gauge. In particular there is exact abelian dominance for the string tension measured with Polyakov loop correlators [2].

A good test of whether the Polyakov gauge projected abelian configurations capture some genuine physics would be to measure the string tension using

time-like Wilson loops and compare this to the string tension obtained with Polyakov loop correlators. Unfortunately this cannot be done directly because the gauge fixing introduces so much noise that one would need a huge number of configurations to get enough statistics.

We can however use an ensemble of smoothed configurations and do all the measurements on them. Since smoothing does not change anything that is genuine long distance physics, this is perfectly justified. In fact we could look at the smoothed configurations as another ensemble generated with some unknown action which produces different short distance structure but its β value is adjusted to be at the same physical lattice spacing (fixed e.g. with the string tension) as the unsmoothed configurations.

We generated an ensemble of $20 \cdot 12^4$ configurations with the fixed point action of Ref. [7] at $\beta = 1.5$ which corresponds to a physical lattice spacing of 0.144 fm. After one smoothing step we measured both the full SU(2) and the Polyakov gauge projected U(1) heavy quark potential on them using time-like Wilson loops. We used the method and computer code of Heller et al. [8]. We computed both on axis and off axis loops and the effective potential for different time extensions of the loops have been obtained as

$$V(R) = -\ln \frac{W(R, T+1)}{W(R, T)}. \quad (1)$$

Our results are shown in Figure 1. In the SU(2) case we have a good plateau at $T = 3$ (this has also been confirmed on another ensemble of larger statistics) but in the U(1) case the potential decreases considerably with increasing T even at this point. Therefore in the SU(2) case we present the $T = 3$ effective potential and for the U(1) case we plot the effective potential with several T values. One can conclude that in the $T \rightarrow \infty$ limit the U(1) string tension is probably very close to zero.

The discrepancy is striking. We would also like to note that the static quark potential measured by Polyakov-loop correlators is by the very definition of the procedure exactly the same as the full non-abelian potential. We also note that the string tension obtained from Polyakov loop correlators and timelike Wilson loops should be the same (up to some small finite size effects). This means that two different but physically equivalent measurements of the same physical quantity give absolutely different results on the Polyakov gauge projected configurations.

For a real test of abelian dominance one has to fix the gauge, do the abelian projection and show that the resulting abelian configurations reproduce all the important long-distance properties of the non-abelian model. For this test to be nontrivial one has to include in the measurement a set of operators large

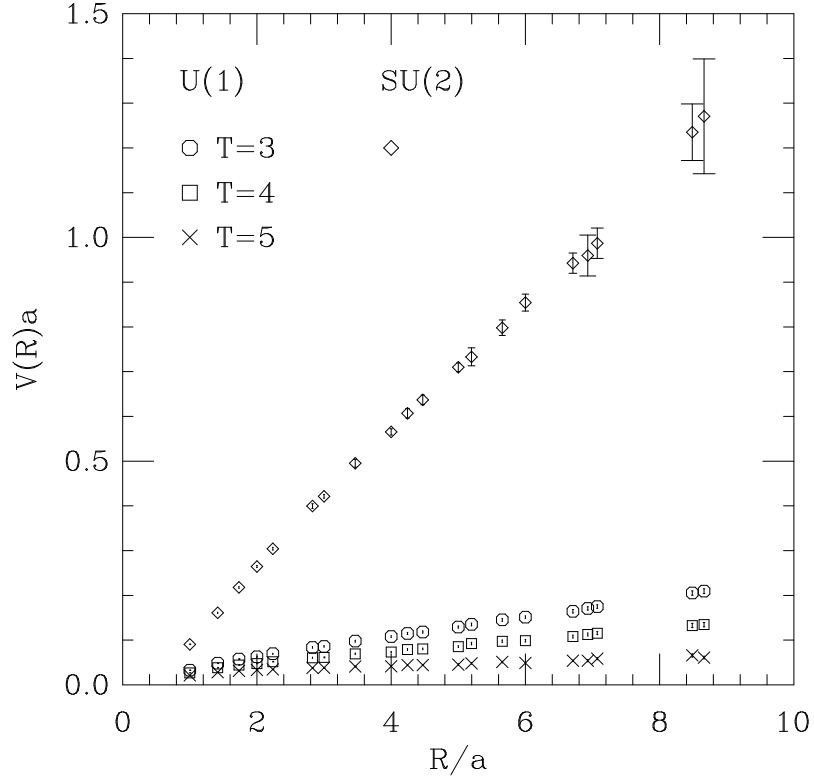


Figure 1: The static quark potential measured with timelike Wilson loops. Diamonds correspond to the full SU(2) potential, the other three symbols represent the U(1) potential measured in the Polyakov gauge with Wilson loops of different time extensions.

enough so that not all of them can be diagonalised at the same time on any non-abelian configuration.

Another important issue about gauge fixing is space-time symmetries. Since we want to choose a particular gauge and not change it when measuring different physical quantities and/or under different physical conditions, it seems reasonable to require that the gauge fixing respect all the space-time symme-

tries of the lattice. Otherwise the abelian projected configurations would break this symmetry and the resulting continuum theory would not be euclidean symmetric unless the symmetry is restored in some miraculous way. This is also strongly suggested by our results concerning the Polyakov gauge which treats the time and space directions differently. Recently Del Debbio et al. [9] also showed that asymmetries in the gauge fixing will lead to similar asymmetries in the abelian projected configurations. They found that if the maximal abelian gauge fixing is done only in a certain plane then abelian dominance holds only for Wilson loops in that plane.

Some people seem to be troubled by the fact that abelian dominance depends on the gauge choice and there are still efforts in the literature to prove the contrary [2]. Our result for the Polyakov gauge strongly suggests that the physics of the abelian projection is not only very strongly gauge dependent but in most of the arbitrarily chosen gauges the abelian projected configurations do not even have a consistent physical meaning. In our opinion it is only natural to expect that abelian observables depend strongly on the gauge fixing and rather than trying to find some gauge independence (even in a limited sense) one should concentrate on finding a *particular* gauge in which the abelian degrees of freedom reproduce as much of the non-abelian dynamics as possible. It is then crucial that the information discarded in the projection be minimised. This is exactly what can be achieved with a suitable gauge fixing. The only well tested method for this is fixing to the maximal abelian gauge (MAG) which minimises the off-diagonal components of the link degrees of freedom, the ones that are discarded in the projection [3]. This is done by maximising the following quantity:

$$G[U] = \sum_l \text{tr}(U_l^\dagger \sigma_3 U_l \sigma_3), \quad (2)$$

where the sum runs over all the links, σ_3 , is a Pauli matrix and U_l is the link SU(2) matrix on l . Geometrically $U_l^\dagger \sigma_3 U_l$ can be visualised as a unit vector in the three-dimensional space spanned by the Pauli matrices. This vector is obtained from σ_3 by applying to it the orthogonal transformation corresponding to U in the adjoint representation of SU(2). Consequently the trace which is summed in eq. (2) is the projection of the rotated σ_3 onto the σ_3 direction. This quantity is maximal when the rotation happens around the σ_3 axis, i.e. U is of the form $\exp(i\alpha\sigma_3/2)$. Maximising $G[U]$ thus results in putting all the link matrices as close as possible to this diagonal form.

There might be other gauge choices that preserve the long distance features better than the MAG but the MAG is the one that – at least locally – puts as much of the fluctuations as possible into the abelian diagonal part of the link variables. For this reason the MAG is a priori a better choice than the gauges that diagonalise an arbitrarily selected set of operators like e.g. the Polyakov

loops.

4 Abelian dominance and short-range fluctuations

In this section we study how abelian dominance in the maximal abelian gauge depends on the precise nature of short distance fluctuations.

We generated 100 $8^3 \times 12$ lattices with the fixed point action of Ref. [7] at $\beta = 1.5$ (lattice spacing $a = 0.144$ fm). At first as a check we verified that abelian dominance holds for this ensemble. We transformed the configurations into the maximal abelian gauge maximising (2). This was done using the usual overrelaxation procedure iterated until the change in the gauge fixing action became less than 10^{-8} per link. After abelian projecting these configurations the heavy quark potential was extracted from time-like Wilson loops in the same way as in the previous section. In Fig. 2 this abelian potential is compared with the SU(2) potential measured on the same ensemble without projection.

A fit to the form

$$V(r) = V_0 - \frac{e}{r} + \sigma r \quad (3)$$

gives $\sigma_{na} = 0.123(7)$ for the non-abelian and $\sigma_{ab} = 0.119(5)$ for the abelian string tension in lattice units.

After this check we applied one step of smoothing to the same ensemble of SU(2) configuration and repeated the measurement of the abelian and non-abelian potential on the smoothed configurations. The results obtained are shown in Fig. 3 and a fit to eq. (3) gives $\sigma_{na} = 0.115(9)$ and $\sigma_{ab} = 0.080(10)$ for the SU(2) and the U(1) string tension respectively.

The SU(2) string tension on the smoothed configurations is essentially the same as on the unsmoothed ones, reflecting the fact that smoothing does not change the long-distance features. On the other hand, as a result of smoothing, the abelian string tension dropped by about 30%. This shows that the abelian string tension is very sensitive to the details of the short-distance fluctuations on the SU(2) configurations.

One smoothing step reduces the fluctuations on the length scale of the lattice spacing. If several smoothing steps are applied successively, larger scale fluctuations are also expected to be gradually washed away. To check how this affects the abelian string tension, we did another three steps of smoothing on the SU(2)

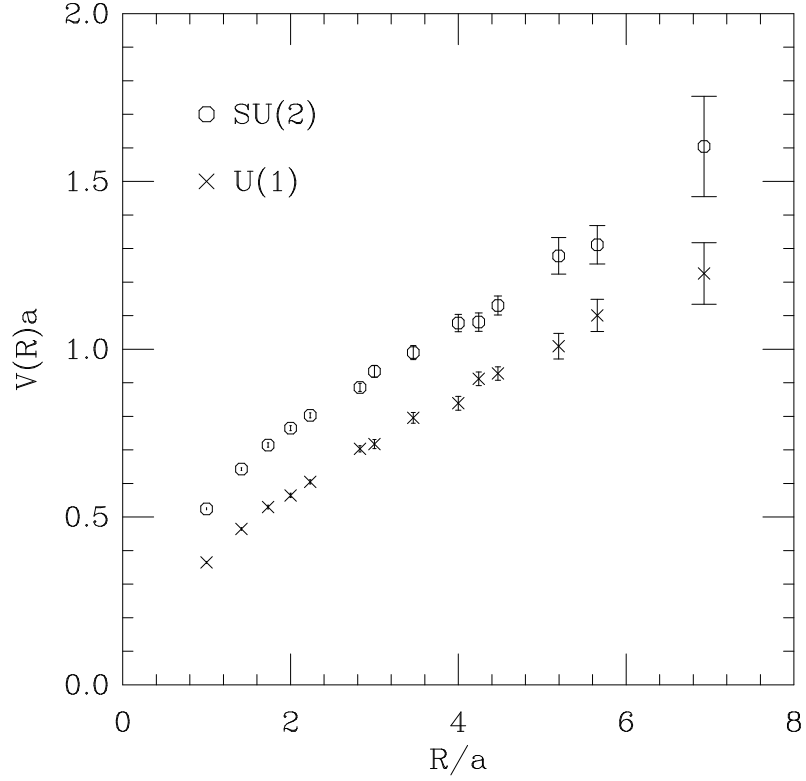


Figure 2: The heavy quark potential measured before (SU(2), octagons) and after (U(1) crosses) abelian projection in the maximal abelian gauge on the original configurations.

configurations and after each step both the abelian and the non-abelian string tension were measured. We found that neither the abelian nor the non-abelian string tension was changed by the additional smoothing steps. The stability of the SU(2) string tension with respect to smoothing was expected but it is rather surprising that while the U(1) string tension changed dramatically in the first step of smoothing, it remained stable after further smoothing. This suggests that the abelian string tension is a delicate combination of the shortest (order a) and longest range fluctuations but it is rather insensitive to intermediate

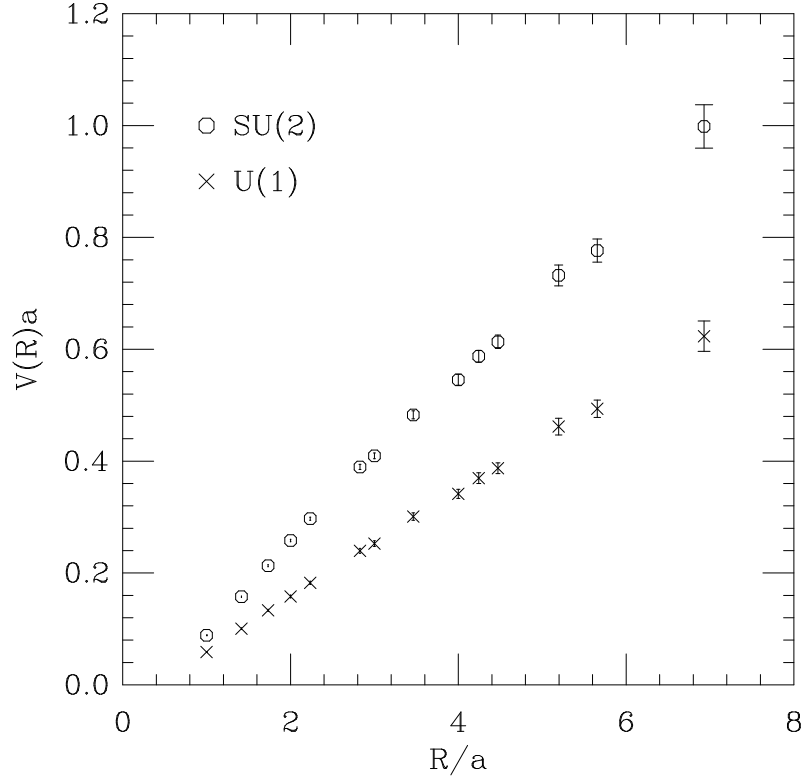


Figure 3: The heavy quark potential measured on once smoothed configurations. Octagons correspond to the SU(2) potential, crosses to the U(1) potential obtained by gauge fixing and projecting the smoothed configurations in the maximal abelian gauge.

length scales. It seems to us quite impossible to reconcile this fact with the expectation that the abelian string tension is a genuine long-distance physical observable which is in some sense equivalent to the SU(2) string tension. In view of this, the approximate abelian dominance found with Wilson action in the maximal abelian gauge seems to be an accident rather than a fundamental physical phenomenon.

5 Conclusions

We used a renormalisation group based smoothing to address two questions related to abelian dominance. Smoothing drastically reduces short distance fluctuations but it preserves the long distance physical properties of the $SU(2)$ configurations. This enabled us to extract the abelian heavy-quark potential from time-like Wilson loops on Polyakov gauge projected configurations. We obtained a very small string tension (probably zero). This is inconsistent with the string tension extracted from Polyakov loop correlators which trivially reproduces the full $SU(2)$ string tension. We then argued on general grounds that the only promising gauge choice in which the abelian projected configurations might capture most of the non-abelian physics, is the maximal abelian gauge.

We also applied the smoothing to test how sensitive abelian dominance in the maximal abelian gauge is to the short distance fluctuations. We found that on smoothed $SU(2)$ configurations the abelian string tension was about 30% smaller than the $SU(2)$ string tension which was unaffected by smoothing. In other words, two ensembles of $SU(2)$ configurations, having the same long distance physical content ($SU(2)$ string tension) differing only in the small scale fluctuations, give different $U(1)$ string tensions. This shows that the abelian string tension is not a genuine long distance observable, it is also very sensitive to the shortest distance scale. If abelian dominance is to be regarded as a fundamental phenomenon, it would be essential to show that it persists in the continuum limit and in this limit it becomes independent of the short distance details of the configurations. Our result suggests that this is quite unlikely to happen.

In the present paper we did not address the role that abelian monopoles might play in the confinement mechanism. In the recent literature there is a lot of work along this line [10] but we feel that the first question one has to ask is whether there is a gauge in which the abelian projection reproduces the essential physical properties of the non-abelian configurations in a consistent way. We think that this question has not been unambiguously answered yet. Until a positive answer to this question is found, abelian monopole condensation cannot be accepted as a serious candidate for explaining confinement.

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